Accelerating cosmologies from compactification with a twist

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It is demonstrated by explicit solutions of the $4+n$–dimensional vacuum Einstein equations that accelerating cosmologies in the Einstein conformal frame can be obtained by a time–dependent compactification of string/M–theory, even in the case that internal dimensions are Ricci–flat, provided one includes one or more geometric twists. Such acceleration is transient. When both compact hyperbolic internal spaces and geometric twists are included, however, the period of accelerated expansion may be made arbitrarily large.

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The observation that the present expansion of the universe is accelerating [1] has proved a challenge to fundamental theories such as string/M–theory, and its low–energy supergravity limits. Fundamental scalar fields are abundant in such higher–dimensional gravity theories, and potentially provide a natural source for the gravitationally repulsive "dark energy" that could explain the present cosmic acceleration, or alternatively at much higher energy scales the very early period of cosmic inflation. However, the constraints imposed by string/M–theory on particular scalar fields, such as the moduli corresponding to 6 or 7 extra compactified dimensions, are such that realistic scenarios giving accelerating cosmologies are very difficult to arrive at.

Indeed for many years, it was assumed that cosmic acceleration was ruled out for supergravity compactifications on the basis of a "no go" theorem [2], which forbids accelerating cosmologies in the presence of static extra dimensions in supergravity compactifications, assuming that one wishes to stay within the realm of classical supergravity rather than resorting to the addition of "quantum correction" terms to the action. Recently Townsend and Wohlfarth [3] demonstrated, however, that it was possible to circumvent the no–go theorem in a time–dependent supergravity compactification using compact extra dimensions with negative curvature. Many additional examples [4]–[10] were subsequently found.

The advent of time–dependent compactifications is an important one, as it has the potential to offer a resolution to the dilemma posed by the observed cosmic acceleration within a natural theoretical framework. Nonetheless, a number of substantial problems remain in the models studied to date. In this Letter we will present new solutions arising from compactifications with a geometrical twist, which we believe overcome these problems to give time–dependent supergravity compactifications with compelling physical features.

Let us turn to the open issues in time–dependent cosmological compactifications. Firstly, in the Townsend–Wohlfarth (henceforth TW) model [3] the no–go theorem was only circumvented by the choice of negatively curved internal spaces, which are made compact by periodic identification. While such spaces have particular features, such as the absence of massless Kaluza–Klein vectors which may have some phenomenological appeal, the restriction on the curvature is a severe one, especially given the view that Ricci–flat internal space compactifications are among the most natural in string theory.

The second important issue is the question of the length of the period of accelerated expansion. The TW model and its successors incorporate scalar fields whose potential energy does not admit a local minimum with positive vacuum energy. A consequence of this is that the period of acceleration is transient, corresponding to just a few e–foldings. Again if one is to apply the model to describe the present cosmic evolution, then this is not a major difficulty but does place some constraints on the parameters involved. However, if rather than a model of the present accelerated cosmological evolution a model of very early universe inflation via time–dependent compactifications was desired, then this would be a significant problem.

In this Letter we will present model cosmologies from time–dependent compactifications which overcome both of the problems above, while also satisfying a third property of theoretical naturalness. The limitations with the models studied to date arise, we believe, from an oversimplification of the effective potentials that arise from compactifications. In particular, when a number of scalars associated with different dimensions are present, their interactions can give rise to effects which are absent if the extra dimensions form a single space of constant curvature, as in the TW model. Product space compactifications have already been considered, sometimes also in the presence of higher dimensional form–fields with non–zero fluxes [8]. However, while higher–dimensional fluxes go a little way towards altering the form of the compact-

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ification potential, for example, by allowing the presence of a local de Sitter minimum [8], in the examples studied to date there is not enough parameter freedom to allow significant periods of accelerated expansion.

Here we will explicitly consider cosmologies that arise in models of gravity which correspond to the dimensional reduction to 4 dimensions of the Einstein equations in 10, 11 or generally $4+n$ dimensions, where some of the extra dimensions form product spaces *with a geometrical twist*. In particular, consider a $(4{+}n)$–dimensional metric *ansatz* with $n = p + 3$ internal dimensions,

$$\mathrm{d}s^2_{4+n} = \mathrm{e}^{-2\Phi}\mathrm{d}s^2(M_4) + r_1^{\,2}\,\mathrm{e}^{2\phi_1}\mathrm{d}s^2(\mathcal{M}^p_{\epsilon_1}) + r_2^{\,2}\,\mathrm{e}^{2\phi_2}\,\mathrm{d}s^2(\mathcal{M}^2_{\epsilon_2}) + r_3^{\,2}\,\mathrm{e}^{2\phi_3}\left(\mathrm{d}z + \varpi_{\epsilon_2}\right)^2, \quad (1)$$

where $\phi_i = \phi_i(u)$, the parameters r_i define appropriate curvature radii, $\mathrm{d}s^2(M_4)$ is the metric of the physical large dimensions in the form

$$\mathrm{d}s^2(M_4) = -a^{2\delta}(u)\mathrm{d}u^2 + a^2(u)\mathrm{d}\Omega^2_{k,3}, \quad (2)$$

and δ is a constant, the choice of which fixes the nature of the time coordinate, u. The internal space $\mathcal{M}^p$ is a p–dimensional space of constant curvature of sign $\epsilon_1 = 0, \pm 1$, and the remaining 3 internal dimensions form a twisted product space, $\mathcal{M}^2 \ltimes \mathcal{M}^1$, as follows

$$\begin{aligned}
\mathrm{d}s^2(\mathcal{M}^2_{+1}) &= \mathrm{d}x^2 + \sin^2 x\,\mathrm{d}y^2, & \varpi_{+1} &= f\cos x\,\mathrm{d}y\ , \\
\mathrm{d}s^2(\mathcal{M}^2_{0}) &= \mathrm{d}x^2 + \mathrm{d}y^2, & \varpi_{0} &= \frac{f}{2}(x\mathrm{d}y - y\mathrm{d}x), \\
\mathrm{d}s^2(\mathcal{M}^2_{-1}) &= \mathrm{d}x^2 + \sinh^2 x\,\mathrm{d}y^2, & \varpi_{-1} &= f\cosh x\,\mathrm{d}y,
\end{aligned} \quad (3)$$

when $\epsilon_2 = +1, 0, -1$ respectively, f being the twist parameter. One chooses the Einstein conformal frame in four dimensions by setting $\Phi = p\phi_1/2 + \phi_2 + \phi_3/2$, and so Newton's constant is time–independent.

To demonstrate the novel features introduced by a geometric twist, we will begin by presenting a special exact solution in the case that the physical universe is spatially flat, $k = 0$, and both $\mathcal{M}^p$ and $\mathcal{M}^2$ are Ricci–flat, i.e., $\epsilon_1 = 0$ and $\epsilon_2 = 0$, but with non–zero twist, f. This solution to the $(p{+}7)$–dimensional vacuum Einstein equations is most readily written down in the gauge, $\delta = 3$, used in Ref. [3]. In particular, we find

$$\begin{aligned}
a(u) &= a_0\,\mathrm{e}^{[(3q+4)c+pc_1]qu/8}\left[\cosh\left(\chi(u-u_0)\right)\right]^{q/4}, \\
\phi_2 &= \frac{-\phi_3}{q} = \tfrac{1}{2}\ln\left[\cosh\left(\chi(u-u_0)\right)\right] + \tfrac{1}{4}(p\,c_1 + 3qc)\,u, \\
\phi_1 &= \frac{2}{p}\ln\left(\frac{f r_3 {a_0}^3}{\chi {r_2}^2}\right) - c_1 u,
\end{aligned} \quad (4)$$

where

$$\chi \equiv \tfrac{1}{2}\sqrt{6pq\,c\,c_1 + 3q(3q+4)c^2 - p[(4-q)p+8]c_1^2/q}. \quad (5)$$

and u_0, c and c_1 are integration constants. We have ignored constants which can be absorbed into the r_i. The parameter q denotes the number of twists, and for the metric (1)–(3) we take $q = 1$ in (4), (5).

For reality of χ we require $\alpha_{1-}c < c_1 < \alpha_{1+}c$, where $\alpha_{1\pm} \equiv [3 \pm \mathrm{sgn}(c)2\sqrt{6(3+7/p)}]/(3p+8)$ when $q = 1$. It is readily seen that the scale factor of the physical universe is monotonic for all values of c_1 in this interval. We therefore restrict our attention to solutions with $c > 0$ which correspond to universes which expand as u increases. Thevalue of u_0 is merely a gauge choice and so we set $u_0 = 0$.

In terms of the physical cosmic time, (i.e., proper time of co–moving observers), $t = \pm\int^u \mathrm{d}\bar{u}\,a^3(\bar{u})$, both the early and late time behaviour of the cosmic scale factor is $a \sim t^{1/3}$. This is similar to the TW solution with two minor differences. Firstly, here the $t \to \infty$ limit corresponds to $u \to \infty$, whereas in the TW solution this limit corresponds to $u \to 0^-$. Secondly, the TW solution decelerates slightly less quickly at late times, with $a \sim t^{n/(n+2)}$. Otherwise, the solution is physically very similar to the TW solution. In particular, since the acceleration parameter (for $q = 1$) is given by

$$\begin{aligned}
a^5\ddot{a} \ &= \ \tfrac{1}{4}\chi^2\left(1 - \tfrac{3}{2}\tanh^2\chi u\right) \\
&\quad -\tfrac{1}{32}\left(7c + p\,c_1\right)\left(4\chi\tanh\chi u + 7c + p\,c_1\right) \quad (6)
\end{aligned}$$

where an overdot denotes differentiation w.r.t. cosmic time, t, it follows that solutions will exhibit a period of transient acceleration provided that $\alpha_{2-}c < c_1 < \alpha_{2+}c$, where $\alpha_{2\pm} \equiv [1 \pm 2\sqrt{3(3+7/p)}]/(5p+12)$. Acceleration occurs on the interval $u_- < u < u_+$, where

$$\tanh(\chi u_\pm) \equiv \frac{-(7c + p\,c_1) \pm 2\sqrt{7c^2 + 2p\,c\,c_1 - p(5p+12)c_1{}^2}}{6\chi}.$$

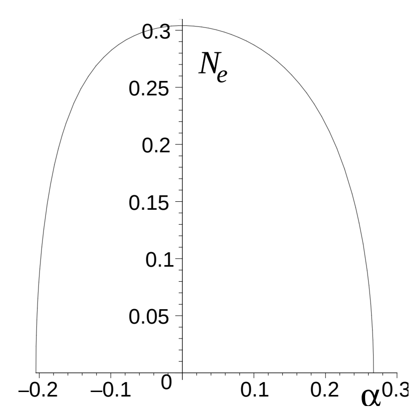

FIG. 1: *The number of e–folds during acceleration epoch* $N_e = \frac{q}{8}\ln\left[\left(\frac{1+\tanh\chi u_+}{1+\tanh\chi u_-}\right)^{\kappa-1}\left(\frac{1-\tanh\chi u_-}{1-\tanh\chi u_+}\right)^{\kappa+1}\right]$ *where* $\kappa \equiv [(3q{+}4)c{+}pc_1]/(2\chi)$, *as a function of the parameter* $\alpha \equiv c_1/c$, *for* $q = 1$, $p = 4$, $d = 11$.

The number of e–folds during the period of acceleration, $N_e \equiv \ln a(u_+) - \ln a(u_-)$ is plotted in Fig. 1. It reaches a maximum $N_e = 0.3041$ at $c_1 = 0$ independently of p, which corresponds to the special case in which the internal space $\mathcal{M}^p$ is static with constant ϕ_1. (The $p = 0$ solution is formally equivalent to (4)–(5) with $c_1 = 0$ and no ϕ_1.) In the general case ϕ_2 slowly increases — giving a slow "decompactification" as in the TW model [3] — while ϕ_3 decreases, and ϕ_1 decreases (increases) if $c_1 u > 0$ ($c_1 u < 0$).

The number of e–folds in the examples is possibly too low to be consistent with observational bounds, especially when one notes that any additional matter obeying the strong energy condition would have a decelerating effect that may decrease the period of transient acceleration. However, the number of e–folds can be increased by increasing the dimension of the twisted space. In particular, consider the case $n = p + 2q + 1$ when the internal space is now the product of a p–dimensional torus $\mathcal{M}^p$, (i.e., $\epsilon_1 = 0$), and the $(2q+1)$–dimensional twisted space

$$
\begin{aligned}
\mathrm{d}s^2_{T^{1,\cdots,1}} &= r_2^2 e^{2\phi_2} \sum_{i=1}^{q} \left(\mathrm{d}x_i^2 + \mathrm{d}y_i^2\right) \\
&+ r_3^2 e^{2\phi_3} \left[\mathrm{d}z + \frac{f}{2} \sum_{i=1}^{q} \left(x_i \mathrm{d}y_i - y_i \mathrm{d}x_i\right)\right]^2, \quad (7)
\end{aligned}
$$

which has topology $(\mathbb{T}^2 \times \cdots \times \mathbb{T}^2) \ltimes S^1$. Eqns (4)–(5) with q arbitrary in fact represent the solution to the $(4+p+2q+1)$–dimensional vacuum Einstein equations with this more general metric ansatz, where now $\Phi = p\phi_1/2 + q\phi_2 + \phi_3/2$. For small values $q > 1$ the qualitative features are the same as the $q = 1$ case, with transient acceleration and a maximum number of e–folds at $c_1 = 0$, independently of p. For example, the maximum number of e–folds for $q = 2$, 3, 4 are respectively $N_e = 0.3689, 0.4019, 0.4221$, a marginal increase.

Next we turn our attention to solutions with a period of acceleration which is not merely transient. In addition to a twist, we will take the curvature of the internal space to be non–zero. To this end, it is convenient to set $\delta = 0$, so that the metric (2) takes the standard Friedmann–Robertson–Walker form, and u becomes the cosmic time, t. For the metric (1), with arbitrary ϵ_i, the field equations may be written in the form

$$
\begin{aligned}
&\ddot{\Phi} + 3H\dot{\Phi} - 2K - 3\frac{\ddot{a}}{a} = 0, \\
&\ddot{\Phi} + 3H\dot{\Phi} - 2\left(\frac{\dot{a}}{a}\right)^2 - \frac{\ddot{a}}{a} - \frac{2k}{a^2} = 0, \qquad (8)\\
&\ddot{\phi}_i + 3H\dot{\phi}_i - \delta_1 V_i - \delta_2 V_F = 0, \qquad i = 1,2,3
\end{aligned}
$$

where $\delta_1 = 2/p$, $\delta_2 = 0$ (for $i = 1$), $\delta_1 = 1$, $\delta_2 = 2$ (for $i = 2$), $\delta_1 = 0$, $\delta_2 = -2$ (for $i = 3$), $H \equiv \dot{a}/a$ is the Hubble parameter, k is the spatial curvature, and the kinetic and potential terms are respectively

$$
\begin{aligned}
K &= \tfrac{p(p+2)}{4}\dot{\phi}_1^2 + 2\dot{\phi}_2^2 + \tfrac{3}{4}\dot{\phi}_3^2 + p\dot{\phi}_1\dot{\phi}_2 + \dot{\phi}_2\dot{\phi}_3 + \tfrac{p}{2}\dot{\phi}_3\dot{\phi}_1, \\
V &= V_1 + V_2 + V_F = \Lambda_1 \mathrm{e}^{-(p+2)\phi_1 - 2\phi_2 - \phi_3} \\
&\quad + \Lambda_2 \mathrm{e}^{-p\phi_1 - 4\phi_2 - \phi_3} + F^2 \mathrm{e}^{-p\phi_1 - 6\phi_2 + \phi_3}, \qquad (9)
\end{aligned}
$$

where $\Lambda_1 = -p(p-1)\epsilon_1/(2r_1^2)$, $\Lambda_2 = -\epsilon_2/r_2^2$ and $F \equiv (f/2)(r_3/r_2^2)$. Even with $\epsilon_i = 0$, the volume modulus field has a non-zero (positive) potential, and so the cosmological solutions with $f > 0$ circumvent the no-go theorem [2], while retaining *Ricci-flat internal spaces.* Many additional examples are given in [11].

In terms of alternative canonically normalized scalars, which may be defined by $\varphi_1 \equiv -\sqrt{\frac{p+5}{p+4}}\,\phi_3$, $\varphi_2 \equiv \frac{\sqrt{p(p+4)}}{2}\left(\phi_1 + \frac{1}{p+4}\phi_3\right)$, $\varphi_3 \equiv \frac{p}{2}\phi_1 + 2\phi_2 + \frac{1}{2}\phi_3$, the field equations are

$$
\begin{aligned}
\ddot{\varphi}_i + 3H\dot{\varphi}_i + \frac{\mathrm{d}V}{\mathrm{d}\varphi_i} &= 0, \qquad (10)\\
\dot{H} + K - ka^{-2} &= 0, \qquad (11)
\end{aligned}
$$

along with the Friedmann (constraint) equation

$$
H^2 = \tfrac{1}{3}\left(K + V\right) - ka^{-2}, \qquad (12)
$$

where $K = \frac{1}{2}\sum_i^3 \dot{\varphi_i}^2$. The resulting scalar potential is

$$
V = \Lambda_1\,\mathrm{e}^{-\varphi_2/\beta - \varphi_3} + \Lambda_2\,\mathrm{e}^{-2\varphi_3} + F^2\,\mathrm{e}^{-\sqrt{5-\beta^2}\varphi_1 + \beta\varphi_2 - 3\varphi_3} \qquad (13)
$$

where $\beta \equiv \sqrt{p}/\sqrt{p+4} < 1$. As above we will confine our attention to $k = 0$ cosmologies. Solutions with no twist, $F^2 = 0$, and compact hyperbolic product spaces, i.e., $\Lambda_i > 0$, have been previously discussed [8], and give transient acceleration without a local minimum in the potential. However, if there is a non–trivial twist and at least one of Λ_i is positive, then it may be possible to find models with a large number of e-foldings. Indeed, if we choose Λ_i such that the potential (13) is strictly non-negative then it always has a minimum with respect to a subset of the φ_i directions.

Even with $k = 0$ the general solution to (10)–(13) is highly non–trivial. To demonstrate the general physical effects, we will take $\Lambda_2 = 0$, so that $\mathcal{M}^2$ is a 2-torus, and specialize to the case in which $d = 11$ (or $p = 4$) and $\varphi_3 = b_1 = \text{const}$. An explicit exact solution can then be found in terms of a new logarithmic time variable τ, defined by $\tau = \int^t \mathrm{d}\bar{t}\,\exp\left[-\varphi_2(\bar{t})/\sqrt{2}\right]$, or $\varphi_2 = \sqrt{2}\ln(\mathrm{d}t/\mathrm{d}\tau)$. The explicit solution is then

$$
\begin{aligned}
&(\ln a)' = \tfrac{1}{\sqrt{12}}\sqrt{V_0}\left(\xi + \xi^{-1}\right), \ \varphi_2' = \tfrac{1}{2}\sqrt{V_0}\left(\xi - \xi^{-1}\right), \\
&\varphi_1 = \varphi_2 + \frac{\sqrt{2}b_0}{3}, \ \xi \equiv \tfrac{\sqrt{6}-1}{\sqrt{5}} \tanh\sqrt{\frac{5V_0}{8}}(\tau - \tau_0), \qquad (14)
\end{aligned}
$$

where $' \equiv \mathrm{d}/\mathrm{d}\tau$, τ_0 is a constant,

$$
V_0 \equiv \Lambda_1 \mathrm{e}^{-b_1} + F^2 \mathrm{e}^{-(b_0 + 3b_1)} = 3F^2 \mathrm{e}^{-(b_0 + 3b_1)}. \qquad (15)
$$

and b_0 is fixed once b_1 is chosen. At late times we find $a \propto t^2$, with the corresponding value of $\omega < -2/3$, where $\omega \equiv (K-V)/(K+V)$. In the limit $\varphi_1 \to \varphi_2$ we find $\phi_1 \to -\phi_2/2$ and $\phi_3 \to -2\phi_2$. Thus two of the extra dimensions associated with the space $\mathcal{M}^2$ may grow with time while other dimensions shrink (or vice versa).

The potential V has a minimum with respect to φ_2 at $\varphi_2^{(0)} = \varphi_1 + (\sqrt{2}/3)\ln(2\Lambda_1/F^2) + 2\sqrt{2}b_0/3$, with

$$V(\varphi_0) = \left(\tfrac{27}{4}\Lambda_1 F^4 \mathrm{e}^{-7b_0}\right)^{1/3} \mathrm{e}^{-\sqrt{2}\varphi_1} = V_{,\varphi_2\varphi_2}(\varphi_0). \quad (16)$$

Since the minimum has the curious feature, special to potentials with $\Lambda_2 = 0$, that $V_{,\varphi_2\varphi_2}(\varphi_0) = V(\varphi_0)$, particle–like states will have a mass $m^2 = V(\varphi_0)$. If this value were to be assigned to the vacuum energy at the present epoch, i.e., $V(\varphi_0) \sim 10^{-120}$ in Planck units, then both the vacuum energy and scalar excitations about the vacuum are ultra–light. The degree to which the model would need to be fine–tuned to achieve such an outcome remains to be investigated.

Another alternative would be to take $V(\varphi_0)$ to have the vastly higher energy scale associated with the epoch of inflation in the very early universe. Let us assume that φ_1 starts initially at $\varphi_1 = \varphi_1^{(0)}$, and that Λ_1 and F both are of order unity (in Planck units), then when $\varphi_1 \sim \varphi_1^{(0)} + 47$ the number of e–folds is $N_e \sim 65$. The actual relation between N_e and the shift in φ_1 can be different, however, depending upon the precise values of the compactification scale, $\sqrt{\Lambda_1}$, and the twist parameter, F.

A point also worth emphasizing is that, especially in a flat universe, the de Sitter stage can be transient. If this is the case in our model, we must allow also φ_3 to roll with t. Different choices of vacua could lead to different asymptotic expansion. More specifically, for the potential (13), when $k=0$ and $p=4$ the late time behaviour of the scale factor is characterized by $a(t) \propto t^\gamma$, where

$$\gamma = \begin{cases} 13/19 & (\Lambda_2 = 0,\ \Lambda_1 \neq 0,\ F^2 \neq 0) \\ 3/5 & (\Lambda_1 = 0,\ \Lambda_2 \neq 0,\ F^2 \neq 0) \\ 3/4 & (F^2 = 0,\ \Lambda_i \neq 0) \\ 7/9 & (\Lambda_i \neq 0,\ F^2 \neq 0). \end{cases} \quad (17)$$

It is expected the contribution of dust or radiation could modify the above asymptotic behaviour.

Let us summarize why we believe that exponential potentials arising from compactifications of supergravity models on a twisted product space of time-varying volume are a potentially significant source for dynamical dark energy at the present epoch. Firstly, Townsend and Wohlfarth [3] argued that the no–go theorem for accelerating cosmologies in supergravity compactifications could only be circumvented by including compact hyperbolic internal spaces, and would not work in the Ricci–flat cases. We have shown on the contrary that the no–go theorem is also circumvented by twisted Ricci–flat spaces by explicit construction of exact solutions with the internal space $\mathbb{T}^p \times (\mathbb{T}^2 \times \cdots \times \mathbb{T}^2) \ltimes S^1$. Since Ricci–flat spaces are natural in the string/M–theory context, this is an important development. Secondly, we have demonstrated that it is possible to construct a metastable de Sitter vacuum in the general framework of [8] by incorporating one or more geometric twists in the internal space.

As mentioned above, some of the extra dimensions may eventually grow large, though slowly, without bound and decompactify, a feature that our transient solutions share with those of the TW model. One might argue that provided that the relative size of the extra dimensions and their rate of change remain sufficiently small as to be consistent with observation over any epoch of physical relevance, then the eschatological consequences of such "decompactifications" are no more severe than those of a universe whose ordinary three dimensions accelerate forever. Nonetheless, given that the natural endpoint of the solutions is a fundamentally higher–dimensional runaway epoch [12], it is clear that we would have additional cosmic coincidence constraints in explaining why the universe appears to have three large spatial dimensions at the present epoch. However, before arriving at any such conclusions it is important to include additional matter, such as pressureless dust. Matter obeying the strong energy condition will certainly affect the length of any transient acceleration, and may also change the evolution of the scalar fields.

Finally, we remark that geometric twists add richness to string/M–theory cosmology, and may potentially lead to the realization of new cosmological scenarios.

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